Analysis of New 4D Hyperchaotic Systems MACM and Implementation Using Runge-kutta

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ABSTRAK

In general, hyperchaotic system is defined as a chaotic system with more than one positive Lyapunov exponent, this implies that its chaotic dynamics extend in several different directions simultaneously. The discovery of reliable and effective methods for solving hyperchaotic systems is carried out extensively. The aim of this research is to implement nonlinear hyperchaotic using runge-kutta. The NHS system is built by inspecting, modifying and adding one state to the MACM chaotic system. In this study, we use the same initial conditions x_0=y_0=z_0=w_0=1 and the parameters a = 2,b = 2,c = 0.5, and d = 14.5. At first we experiments modified approaches, namely Classic Runge-Kutta Method 3 Order, Runge-Kutta Method Based On Arithmetic Mean, Metode Runge-Kutta Method Based On Geometric Mean, Modified Runge-Kutta Geometric Mean Based Method-1, RKLCM Approaches Instead of RKGM using MAPLE software as a calculation tool. In this work, we observe that the Runge-Kutta Method Based on Arithmetic Mean has the smallest error value with an average variable x of 0.000606953, variable y of 0.009280181, variable z of 0.000378639 and variable w of 0.60781585. Then also in RKLCM Approaches Instead of RKGM it has the largest error value with an average variable x of 0.483789793, variable y of 0.620803849, variable z of 0.92097139, and variable w of 0.596137549.

I. Pendahuluan

In general, hyperchaotic system is defined as a chaotic system with more than one positive Lyapunov exponent, this implies that its chaotic dynamics extend in several different directions simultaneously [1]. Therefore, comparing with the traditional chaotic system, hyperchaotic system has more complex dynamical behaviors which can be used to enhance the security of chaotic communication system. Consequently, the topic of theoretical design and circuitry realization of various hyperchaotic systems has recently become hotspot in the nonlinear research field [2-3]. Hyperchaos has been found numerically and experimentally by adding a simple state feedback controller to Chua's circuit [4], Chen system [5], or Lorenz equation [6].

In recent years, chaos systems have become the subject of many studies in the fields of science and engineering. A large number of new chaotic systems have been proposed one after another, and their application scopes are more and more extensive [7]. With the progress of science and technology, chaos has been applied not only to secure communications [6], image processing [8], experimental network synchronization [9], electronic circuits [10], technology as fingerprint encryption [11], digital cryptosystems based on chaos [12]. This is because chaotic signal has good pseudorandom, initial-value sensitive, and long-term unpredictable characteristics, which enhances the confusion and diffusion of encrypted data.

The mathematical definition of a hyperchaotic system is based on a chaotic system with more than one positive Lyapunov exponent, and simultaneously its dynamics are richer and more extended in the phase plane [13]. The hyperchaos has more complex dynamical behaviors than the chaotic system, and it can be found coupling k chaotic systems.

The discovery of reliable and effective methods for solving hyperchaotic systems is carried out extensively. The NHS system is built by inspecting, modifying and adding one state to the MACM chaotic system [14]. The proposed NHS is described as follows

$$\begin{aligned} x &= -ax - byz \\ y &= -x + cy + cw \end{aligned} \quad (1)$$

$$z = d - y2 - z$$
$$w = x - w$$

In this work, we firstly introduce a new 4D hyperchaotic system (NHS) via modifying 3D MACM system inspired by the above works [15]. Two critical parameters, only two nonlinearities, flexible and robust, one unstable equilibrium saddle-point, and low-cost electronic implementation for the continuous and discretized versions are the main novelties of the proposed hyperchaotic system. All these features result in a highly attracting digital implementation, such as low complexity time, high iterations per second (ips) where chaos is preserved, and it may be of great interest for engineering applications such as chaos-based cryptography, biometric systems, telemedicine, and secure communications. According to our best knowledge, the digital implementation of the nonlinear hyperchaotic using RK3.

II. Material dan Metode

A. MACM chaotic system

First, we consider the Lorenz system, which is a well known example of a CS. Lorenz's three-variable model provides a practical test case with qualitatively realistic properties [16]; it is represented by the nonlinear state equations described as:

$$\begin{aligned} x &= \sigma\,(y - x), \\ y &= rx - xz - y, \\ y &= xz - bz, \end{aligned} \tag{2}$$

where x, y, and z are the state variables and the standard parameter values for Lorenz's chaotic attractor are $\sigma = 10\sigma = 10$, $r = 28r = 28$, and $b = 8/3b = 8/3$. The MACM CS was proposed in 2017 [17], which is described by

$$\begin{aligned} x &= -ax - byz, \\ y &= -x + cy, \\ z &= d - y^2 - z, \end{aligned} \tag{3}$$

where x, y, and z are the state variables and the MACM system presents chaotic behavior for the following parameter values: $a = b = 2a = b = 2$, $c = 0.5c = 0.5$, and $d = 4d = 4$. In this section, state equations and tests to verify the hyperchaos existence are presented. The NHS system is built by inspecting, modifying, and adding one state to the MACM chaotic system [18]. The proposed NHS is described as follows

$$\begin{aligned} x &= -ax - byz \\ y &= -x + cy + cw \\ z &= d - y2 - z \\ w &= x - w \end{aligned} \tag{4}$$

The system (4) has ten terms, two quadratic nonlinearities, and four parameters $a, b, c, d \in R+$ $a, b, c, d \in R+$, with $c < a + 2c < a + 2$, where b and d are characterized as the bifurcation parameters. The nonlinear NHS (1) is hyperchaotic with $a = 2, b = 2, c = 0.5, a = 2, b = 2, c = 0.5,$ and $d = 14.5. d = 14.5.$ In this study, we use the same initial conditions $x_0 = y_0 = z_0 = w_0 = 1$ $x_0 = y_0 = z_0 = w_0 = 1$ and the parameters $a = 2, b = 2, c = 0.5, a = 2, b = 2, c = 0.5,$ and $d = 14.5. d = 14.5.$

B. Numerical Algorithm

The non-linear functions 4D MACM in their DVs describe the states x, y, z,and w respectively. RK3 algorithm is one of the most widely used methods for solving differential equations [19]. define the classical Runge-Kutta third order method as follows

$$\begin{aligned} k_1 &= f(x_n, y_n) \\ k_2 &= f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}k_1\right) \\ k_3 &= f(x_n + h, y_n + hk_1 + hk_2) \end{aligned} \tag{5}$$

and

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 4k_2 + k_3) \tag{6}$$

and make an adjustment of the parameters ai, i = 1,2,3 to attain third order accuracy. Now equation (1.4) can be written also as

$$y_{n+1} = y_n + \frac{h}{4}(k_1 + 2k_2 + k_3) \tag{7}$$

The proposed reformulation equations are given below.

$$y_{n+1} = y_n + \frac{h}{2}\left(\frac{(k_1 + k_2}{2} + \frac{(k_2 + k_3)}{2}\right) \tag{8}$$

C. Method Analysis

RK3 algorithm is one of the most widely used methods for solving differential equations [20]. The NA of RK3 is given by

$$\begin{aligned} x_{n+1} &= x_n + \frac{h}{2}\left(\frac{(k_1 + k_2}{2} + \frac{(k_2 + k_3)}{2}\right) \\ y_{n+1} &= y_n + \frac{h}{2}\left(\frac{(k_1 + k_2}{2} + \frac{(k_2 + k_3)}{2}\right) \\ z_{n+1} &= z_n + \frac{h}{2}\left(\frac{(k_1 + k_2}{2} + \frac{(k_2 + k_3)}{2}\right) \\ w_{n+1} &= w_n + \frac{h}{2}\left(\frac{(k_1 + k_2}{2} + \frac{(k_2 + k_3)}{2}\right) \end{aligned} \tag{9}$$

where $k_1, k_2, k_3 k_1, k_2, k_3$ and $k_4 k_4$ are referred to as coefficients of the first equation, similarly, the parameters $l_1, l_2, l_3 l_1, l_2, l_3$ and $l_4 l_4$ are referred to as coefficients of the second equation, and the parameters m_1, m_2, m_3 m_1, m_2, m_3, and $m_4 m_4$ are referred to as coefficients of the third equation, and the parameters $m_1, m_2, m_3 n_1, n_2, n_3 m_1, m_2, m_3 n_1, n_2, n_3$, and $n_4 n_4$ are referred to as coefficients of the fourth equation. The coefficients described in system (9) are given by

$$\begin{aligned} k_1 &= f1(x_{(n)} y_{(n)}, z_{(n)}, w_{(n)}) \\ l_1 &= f2(x_{(n)} y_{(n)}, z_{(n)}, w_{(n)}) \\ m_1 &= f3(x_{(n)} y_{(n)}, z_{(n)}, w_{(n)}) \\ n_1 &= f4(x_{(n)}, y_{(n)}, z_{(n)}, w_{(n)}) \end{aligned} \tag{10}$$

$$\begin{aligned} k_2 &= f1\left(x_{(n)} + \frac{1}{2}k_1, y_{(n)} + \frac{1}{2}k_1, z_{(n)} + \frac{1}{2}k_1, w_{(n)} + \frac{1}{2}k_1\right), \\ l_2 &= f2\left(x_{(n)} + \frac{1}{2}l_1, y_{(n)} + \frac{1}{2}l_1, z_{(n)} + \frac{1}{2}l_1, w_{(n)} + \frac{1}{2}l_1\right), \\ m_2 &= f3\left(x_{(n)} + \frac{1}{2}m_1, y_{(n)} + \frac{1}{2}m_1, z_{(n)} + \frac{1}{2}m_1, w_{(n)} + \frac{1}{2}m_1\right), \\ n_2 &= f4\left(x_{(n)} + \frac{1}{2}n_1, y_{(n)} + \frac{1}{2}n_1, z_{(n)} + \frac{1}{2}n_1, w_{(n)} + \frac{1}{2}n_1\right) \end{aligned} \tag{11}$$

$$\begin{aligned} k_3 &= f1(x_{(n)} - k_1 + k_2, y_{(n)} - k_1 + k_2, z_{(n)} - k_1 + k_2, w_{(n)} - k_1 + k_2), \\ l_3 &= f2(x_{(n)} - l_1 + l_2, y_{(n)} - l_1 + l_2, z_{(n)} - l_1 + l_2, w_{(n)} - l_1 + l_2), \end{aligned} \tag{12}$$

$$m_3 = f3\left(x_{(n)} - m_1 + m_2, y_{(n)} - m_1 + m_2, z_{(n)} - m_1 + m_2, w_{(n)} - m_1 + m_2\right),$$
$$n_3 = f4\left(x_{(n)} - n_1 + n_2, y_{(n)} - n_1 + n_2, z_{(n)} - n_1 + n_2, w_{(n)} - n_1 + n_2\right)$$

III. Hasil dan Pembahasan

For experimental study we consider the following problems.

$$x = -ax - byz$$
$$y = -x + cy + cw$$
$$z = d - y2 - z$$
$$w = x - w$$

In this study, we use the same initial conditions $x_0 = y_0 = z_0 = w_0 = 1x_0 = y_0 = z_0 = w_0 = 1$ and the parameters $a = 2, b = 2, c = 0.5, a = 2, b = 2, c = 0.5,$ and $d = 14.5 . d = 14.5.$ At first we have performed experiments to verify the robustness of the existing modified approaches, namely RKK3, RKAM, RKGM, MRKLCM1 and RKLCM of RKGM using MAPLE software as a calculation tool.

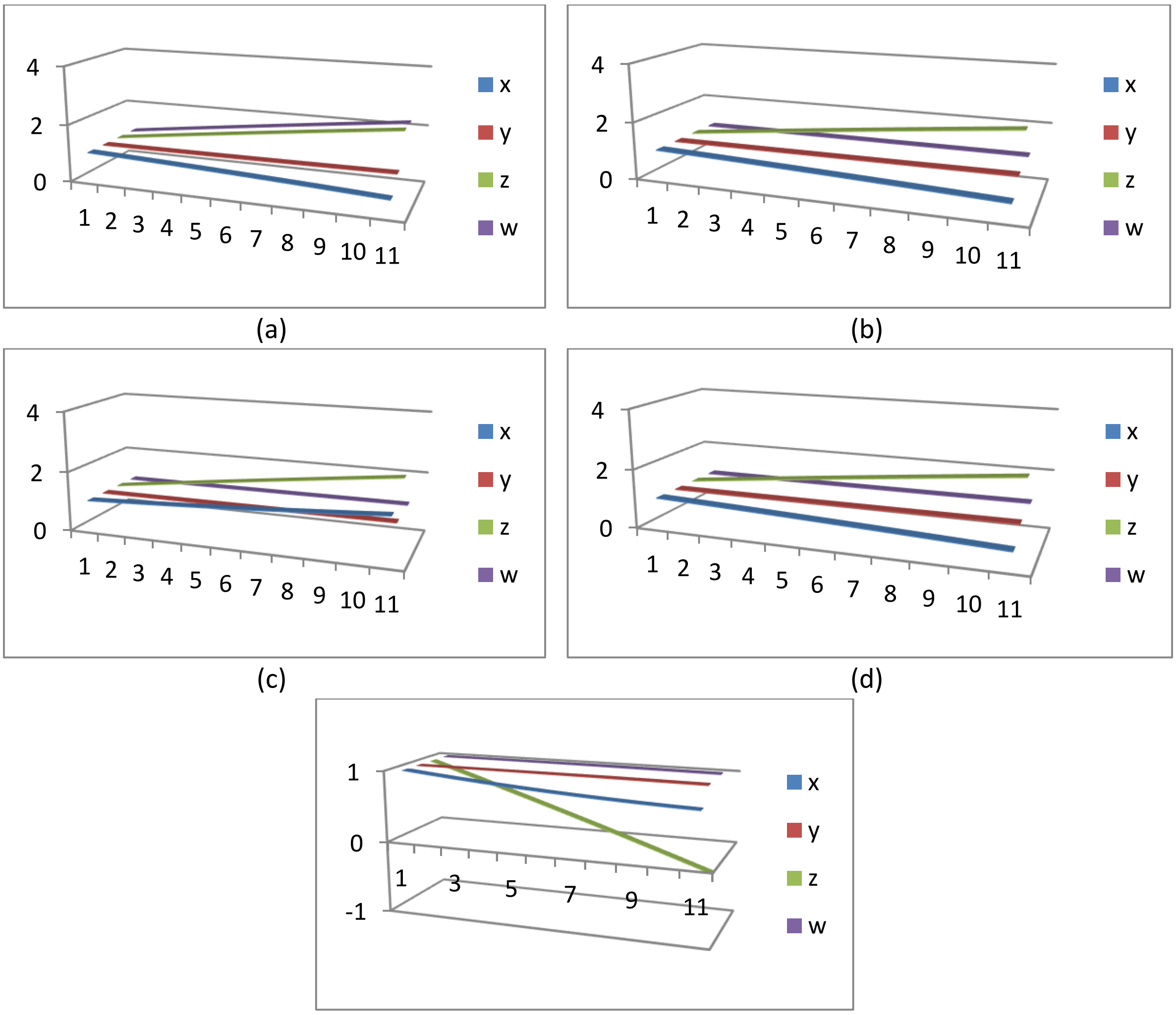

Figure 1. Solution of 3-dimensional variables from the modification method for x, y, z, and w at intervals $0 \le t \le 0.10 \le t \le 0.1$, h=0.01 : (a) Classic Runge-Kutta Method 3 Order; (b) Runge-Kutta Method Based On Arithmetic Mean; (c) Metode Runge-Kutta Method Based On Geometric Mean; (d) Modified Runge-Kutta Geometric Mean Based Method-1; (e) RKLCM Approaches Instead of RKGM;

Table 1. The solution of the modified method for x, y, z, and w on the interval $0 \leq t \leq 0.10 \leq t \leq 0.1$, h=0.01 : (a) Classic Runge-Kutta Method 3 Order; (b) Runge-Kutta Method Based On Arithmetic Mean; (c) Metode Runge-Kutta Method Based On Geometric Mean; (d) Modified Runge-Kutta Geometric Mean Based Method-1; (e) RKLCM Approaches Instead of RKGM;

T	Δ x	Δ y	Δ z	Δ w
0	1	1	1	1
0.01	0.960578	1	1.124057	1.124057
0.02	0.919512	1.001015	1.246884	1.246884
0.03	0.876835	1.003059	1.36847	1.36847
0.04	0.832574	1.006148	1.488809	1.488809
0.05	0.786744	1.010297	1.60789	1.60789
0.06	0.739355	1.015521	1.72503	1.725703
0.07	0.69041	1.021833	1.84224	1.84224
0.08	0.639902	1.02925	1.957489	1.957489
0.09	0.587818	1.037784	2.07144	2.07144
0.1	0.534139	1.047452	2.184081	2.184081

(a)

T	Δ x	Δ y	Δ z	Δ w
0	1	1	1	1
0.01	0.960578	1	1.124057	1
0.02	0.919512	1.00094	1.246884	0.999606
0.03	0.876851	1.001199	1.368483	0.998805
0.04	0.832638	1.002431	1.488859	0.997585
0.05	0.786914	1.004104	1.608016	0.995936
0.06	0.739714	1.006235	1.725956	0.993846
0.07	0.691069	1.008839	1.842682	0.991304
0.08	0.641006	1.011929	1.958196	0.988302
0.09	0.589548	1.01552	2.0725	0.984829
0.1	0.536713	1.019626	2.185596	0.980876

(b)

T	Δ x	Δ y	Δ z	Δ w
0	1	1	1	1
0.01	1.038834	1	1.123093	1
0.02	1.080762	1.000388	1.244975	1.000388
0.03	1.125822	1.001192	1.365649	1.001192
0.04	1.174058	1.002438	1.485118	1.002438
0.05	1.225518	1.004155	1.603385	1.004155
0.06	1.280252	1.006368	1.720452	1.006368
0.07	1.338318	1.009107	1.83632	1.009107
0.08	1.399778	1.012399	1.95099	1.012399
0.09	1.464701	1.016273	2.064463	1.016273
0.1	1.533158	1.020757	2.176737	1.020757

(c)

t	Δ x	Δ y	Δ z	Δ w
0	1	1	1	1
0.01	0.961205	1	1.123059	1
0.02	0.920828	1.000388	1.244908	1.000388
0.03	0.878917	1.001184	1.365549	1.001184
0.04	0.835516	1.002406	1.484986	1.002406
0.05	0.790666	1.004075	1.603223	1.004075
0.06	0.744401	1.006209	1.720261	1.006209
0.07	0.696753	1.008827	1.836102	1.008827
0.08	0.647747	1.011948	1.950749	1.011948
0.09	0.597407	1.01559	2.064202	1.01559
0.1	0.545749	1.019772	2.176461	1.019772

(d)

T	Δ x	Δ y	Δ z	Δ w
0	1	1	1	1
0.01	0.961205	1	0.876941	1
0.02	0.925504	0.999612	0.75267	0.999612
0.03	0.892874	0.998871	0.627169	0.998871
0.04	0.863288	0.997811	0.500417	0.997811
0.05	0.836719	0.996466	0.372396	0.996466
0.06	0.813137	0.994868	0.243086	0.994868
0.07	0.792512	0.993051	0.112471	0.993051
0.08	0.77481	0.991046	-0.01947	0.991046
0.09	0.759998	0.988883	-0.15275	0.988883
0.1	0.748044	0.986594	-0.28738	0.986594

(e)

Figure 1 and Table 1 shows the results of each numerical solution used, such as classic Rungekutta, Runge-Kutta Method Based On Arithmetic Mean, Metode Runge-Kutta Method Based On Geometric Mean, Modified Runge-Kutta Geometric Me $x_0 = y_0 = z_0 = w_0 = 1 x_0 = y_0 = z_0 = w_0 = 1$ of RKGM with initial $a = 2, b = 2, c = 0.5, a = 2, b = 2, c = 0.5,$ $d = 14.5. d = 14.5.$ and the parameters and It can also be seen that the solution variable x in each method modification (b,c,d,e) is greater than the 3rd order classical Runge-kutta (a), variables y and w in each method modification (b,c,d,e) is greater than the classical runge-kutta of order 3(a) and only in the Runge-Kutta Method Based On Arithmetic Mean(b) the z variable is greater than the runge-kutta of order 3(a).

Figure 3. Solusi galat dari metode modifikasi untuk x, y, z, dan w pada interval $0 \leq t \leq 0.10 \leq t \leq 0.1$, h=0.01 : (a) Classic Runge-Kutta Method 3 Order; (b) Runge-Kutta Method Based On Arithmetic Mean; (c) Metode Runge-Kutta Method Based On Geometric Mean; (d) Modified Runge-Kutta Geometric Mean Based Method-1; (e) RKLCM Approaches Instead of RKGM;

t	Δ x	Δ y	Δ z	Δ w
0	0	0	0	0
0.01	0	0	0	1.24E-01
0.02	0	6.20E-04	0	2.47E-01
0.03	1.51E-05	1.86E-03	1.27E-05	3.70E-01
0.04	6.43E-05	3.72E-03	5.06E-05	4.91E-01
0.05	1.70E-04	6.19E-03	1.26E-04	6.12E-01
0.06	3.59E-04	9.29E-03	2.52E-04	7.32E-01
0.07	6.59E-04	1.30E-02	4.42E-04	8.51E-01
0.08	1.10E-03	1.73E-02	7.06E-04	9.69E-01
0.09	1.73E-03	2.23E-02	1.06E-03	1.09E+00
0.1	2.57E-03	2.78E-02	1.52E-03	1.20E+00

(a)

t	Δ x	Δ y	Δ z	Δ w
0	0	0	0	0
0.01	7.83E-02	0	9.64E-04	1.24E-01
0.02	1.61E-01	6.26E-04	1.91E-03	2.46E-01
0.03	2.49E-01	1.87E-03	2.82E-03	3.67E-01
0.04	3.41E-01	3.71E-03	3.69E-03	4.86E-01
0.05	4.39E-01	6.14E-03	4.50E-03	6.04E-01
0.06	5.41E-01	9.15E-03	5.25E-03	7.19E-01
0.07	6.48E-01	1.27E-02	5.92E-03	8.33E-01
0.08	7.60E-01	1.69E-02	6.50E-03	9.45E-01
0.09	8.77E-01	2.15E-02	6.98E-03	1.06E+00
0.1	9.99E-01	2.67E-02	7.34E-03	1.16E+00

(b)

t	Δ x	Δ y	Δ z	Δ w
0	0	0	0	0
0.01	6.27E-04	0	9.98E-04	1.24E-01
0.02	1.32E-03	6.27E-04	1.98E-03	2.46E-01
0.03	2.08E-03	1.88E-03	2.92E-03	3.67E-01
0.04	2.94E-03	3.74E-03	3.82E-03	4.86E-01
0.05	3.92E-03	6.22E-03	4.67E-03	6.04E-01
0.06	5.05E-03	9.31E-03	5.44E-03	7.19E-01
0.07	6.34E-03	1.30E-02	6.14E-03	8.33E-01
0.08	7.85E-03	1.73E-02	6.74E-03	9.46E-01
0.09	9.59E-03	2.22E-02	7.24E-03	1.06E+00
0.1	1.16E-02	2.77E-02	7.62E-03	1.16E+00

(c)

t	Δ x	Δ y	Δ z	Δ w
0	0	0	0	0
0.01	6.27E-04	0	2.47E-01	1.24E-01
0.02	5.99E-03	1.40E-03	4.94E-01	2.47E-01
0.03	1.60E-02	4.19E-03	7.41E-01	3.70E-01
0.04	3.07E-02	8.34E-03	9.88E-01	4.91E-01
0.05	5.00E-02	1.38E-02	1.24E+00	6.11E-01
0.06	7.38E-02	2.07E-02	1.48E+00	7.31E-01
0.07	1.02E-01	2.88E-02	1.73E+00	8.49E-01
0.08	1.35E-01	3.82E-02	1.98E+00	9.66E-01
0.09	1.72E-01	4.89E-02	2.22E+00	1.08E+00
0.1	2.14E-01	6.09E-02	2.47E+00	1.20E+00

(d)

In Table 2, we observe that the Runge-Kutta Method Based on Arithmetic Mean has the smallest error value with an average variable x of 0.000606953, variable y of 0.009280181, variable z of 0.000378639 and variable w of 0.60781585. Then followed by Modified Runge-Kutta Geometric Mean Based Method-1 with an error value on the average variable x is 0.153352453, variable y is 0.202914701, variable z is 0.299737533 and variable w is 0.558334686. Then followed by Modified Metode Runge-Kutta Method Based On Geometric Mean with an error value on the average variable x is 0.267783679, variable y is 0.202701451, variable z is

0.299539413 and variable w is 0.567647402. Then also in RKLCM Approaches Instead of RKGM it has the largest error value with an average variable x of 0.483789793, variable y of 0.620803849, variable z of 0.92097139, and variable w of 0.596137549.

IV. Kesimpulan

This work presented analytical and numerical studies of a new hyperchaotic system (NHS) that produces chaos and hyperchaos, varying only two parameters using Runge-Kutta. The results showed that new hyperchaotic attractor presents two positive Lyapunov exponents, an unstable equilibrium saddle-point at the origin, and it is flexible and robust which allows obtaining different hyperchaotic and chaotic behavior. Figure 1 and 2 shows the results of each numerical solution used, such as classic Rungekutta, Runge-Kutta Method Based On Arithmetic Mean, Metode Runge-Kutta Method Based On Geometric Mean, Modified Runge-Kutta Geometric Mean Based Method-1, RKLCM Approaches Instead of RKGM with initial conditions same initial conditions $x_0 = y_0 = z_0 = w_0 = 1 x_0 = y_0 = z_0 = w_0 = 1$ and the parameters $a = 2, b = 2, c = 0.5, a = 2, b = 2, c = 0.5,$ and $d = 14.5. d = 14.5.$ It can also be seen that the solution variable x in each method modification (b,c,d,e) is greater than the 3rd order classical Runge-kutta (a), variables y and w in each method modification (b,c,d,e) is greater than the classical runge-kutta of order 3(a) and only in the Runge-Kutta Method Based On Arithmetic Mean(b) the z variable is greater than the runge-kutta of order 3(a). In Figure 3, we observe that the Runge-Kutta Method Based on Arithmetic Mean has the smallest error value with an average variable x of 0.000606953, variable y of 0.009280181, variable z of 0.000378639 and variable w of 0.60781585. Then followed by Modified Runge-Kutta Geometric Mean Based Method-1 with an error value on the average variable x is 0.153352453, variable y is 0.202914701, variable z is 0.299737533 and variable w is 0.558334686. Then followed by Modified Metode Runge-Kutta Method Based On Geometric Mean with an error value on the average variable x is 0.267783679, variable y is 0.202701451, variable z is 0.299539413 and variable w is 0.567647402. Then also in RKLCM Approaches Instead of RKGM it has the largest error value with an average variable x of 0.483789793, variable y of 0.620803849, variable z of 0.92097139, and variable w of 0.596137549.

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